9.5 Solving Equations Using Mental Math

YOU WILL NEED
• toothpicks

GOAL
Solve a problem by solving a related equation.

LEARN ABOUT the Math

Megan and Oshana are building this toothpick pattern. They have 28 toothpicks.

What is the largest figure they can build?

A. Write a pattern rule for the number of toothpicks in figure $n$.
B. Write an equation to represent the problem.
C. Your equation may look like $\square \times \text{variable} + \square = 28$. What is the value of the $\square \times \text{variable}$ part?
D. What is the value of your variable?
E. Which figure has 28 toothpicks?
F. Check your solution to the equation.

equation
a statement that two quantities or expressions are equivalent; for example, $4 + 2 = 6$ and $6x + 2 = 14$

solution to an equation
a value of a variable that makes an equation true; for example, the solution to $6x + 2 = 14$ is $x = 2$
Reflecting

**G.** How are your pattern rule in part A and your equation in part B alike? How are they different?

**H.** Think about your equation. What meaning does each part of the equation have when you connect it to the problem?

**I.** How did you use mental math to calculate the solution?

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**WORK WITH the Math**

**Example 1 | Solving a problem using an equation**

Irene breeds and sells rabbits. One day, she sold half of her rabbits. Then, she sold one more. She still had 13 rabbits. How many rabbits did she have to start?

**Denis’s Solution**

I used \( r \) to represent the number of rabbits she started with.

\[
\frac{r}{2} - 1 = 13 \\
\frac{r}{2} = 14 \\
\]

If I subtract 1 from something and get 13, I must have started with 14. This means that \( \frac{r}{2} \) must equal 14.

\[
r = 28 \\
\]

Verify the solution:

Suppose that Irene started with 28 rabbits. If she sold half, she would have 14 left. If she sold 1 more, she would have 13. This matches what the problem said.

The solution \( r = 28 \) is correct. Irene started with 28 rabbits.
Example 2 | Solving a sharing problem using an equation

Scott and his 4 friends mowed lawns together last summer. They split their earnings equally, and each person received $75. Represent the situation with an equation. Determine the total amount of money they earned using your equation.

Nayana’s Solution

\[ \frac{m}{5} = 75 \]

\[ m = 5 \times 75 \]

\[ m = 375 \]

The friends earned $375.

I used \( m \) to represent the money that was shared. The friends divided the money 5 ways. The 5 friends each got $75, so the total amount is 5 times $75.

\[ \frac{x}{3} = 9 \]

\[ 22 = m + 2 \]

\[ \frac{w}{10} = 20 \]

\[ 35 = n - 5 \]

A Checking

   a) \( n + 6 = 13 \)  
   c) \( 9p = 63 \)  
   e) \( \frac{x}{3} = 9 \)
   b) \( w - 11 = 22 \)  
   d) \( 2n + 3 = 15 \)  
   f) \( \frac{y}{4} = 7 \)

2. Jamie wants to know which figure in this pattern has exactly 43 tiles.

   figure 1  
   figure 2  
   figure 3

   a) Write an equation to represent Jamie’s problem.
   b) Solve the equation.
   c) Check your solution. Show what you did.

B Practising

3. Solve each equation.
   a) \( 7b = 84 \)  
   c) \( 8 + z = 30 \)  
   e) \( 22 = m + 2 \)
   b) \( 11 = q - 4 \)  
   d) \( \frac{w}{10} = 20 \)  
   f) \( 35 = n - 5 \)
4. Solve each equation.
   a) $23 = 2m + 3$
   b) $42 = 6a - 6$
   c) $9n - 4 = 32$
   d) $5n + 5 = 40$

5. a) Explain each step in this solution.
   
   \[ 6 + 5m = 16 \]
   Step 1: \[ 5m = 10 \]
   Step 2: \[ m = 2 \]
   
   b) How can you verify that the solution $m = 2$ is correct?


7. a) Write a pattern rule to represent the number of tiles in each figure in this pattern.

   \begin{figure}
   \centering
   \includegraphics[width=0.8\textwidth]{figures.png}
   \caption{Figure 1, Figure 2, Figure 3, Figure 4}
   \end{figure}

   b) Albert wants to build the figure with 24 tiles in this pattern. Write an equation you can solve to determine the number of the figure with 24 tiles.
   
   c) Solve your equation.
   
   d) Verify your solution by drawing the figure and counting the tiles.
   
   e) Determine which figure in this pattern has 21 tiles, using your equation. Describe the figure with 21 tiles.

8. a) Loretta wants to build the figure with 28 pattern blocks in the following pattern. Write an equation you can solve for the number of this figure.

   \begin{figure}
   \centering
   \includegraphics[width=0.8\textwidth]{figures2.png}
   \caption{Figure 1, Figure 2, Figure 3}
   \end{figure}

   b) Solve your equation and verify your solution.
9. **a)** Wilmer wants to build the figure with exactly 30 tiles in the pattern at the left. Write an equation you can solve for the number of the figure.

**b)** Solve your equation and verify your solution.

10. To rent a movie, a company charges $2, plus $1 per day.

**a)** Write a pattern rule to represent the cost, \( c \), of renting a movie for \( d \) days.

**b)** Write an equation to answer the question, “If you have $9, for how many days can you rent a movie?”

**c)** Solve your equation and verify your solution.

11. Susan has $25. She is going to spend $4 on a book, and then $3 per day on lunch.

**a)** Write an equation to answer the question, “For how many days can Susan buy lunch?”

**b)** Solve your equation and verify your solution.

12. Kevin says to Zach, “I am thinking of a number. If you double it and then subtract 1, the result is 7.”

**a)** Write an equation to determine Kevin’s number.

**b)** What steps can you use to solve the equation? Explain.

13. What is the greatest number of squares you can build with 100 toothpicks using this pattern?

14. What is the greatest number of triangles you can build with 100 toothpicks using this pattern?

15. Look at the linear relation \( \square x + \bigcirc = \bigdiamond \). Suppose that you know the value each geometric symbol represents. What steps would you use to solve for \( x \) using mental math?